PHYSICS DEPARTMENT
MINOR I, PHL-110, FIELD AND WAVES (September 6, 2004)

Max. Marks: 25
Time: 1hour

Note: Attempt all questions:

1. (i) A thick metal sheet occupies half space ($x < 0$). The $x > 0$ space is vacuum. A charge Q is placed at (d,0,0). What would be the direction of E at the metal surface at point (0, 2d, 0)? **1**

(ii) A charged particle of charge Q and mass m is initially at rest with height $y = h$ above the ground. It falls under gravity ($-g\ \hat{y}$) and a static magnetic field $B_0\ \hat{z}$. What would be the kinetic energy KE of the particle when it touches the ground? **1**

(iii) A metallic sphere of radius R, carrying a charge Q is surrounded by a thick spherical concentric metallic shell with inner radius as 'a' and outer radius as 'b'. If the shell carries no net charge, what is the surface charge densities at $r = a$, b and R. **1**

(iv) Consider a closed hemispherical surface of radius R and with its base lying in x-y plane. If it lies in a uniform magnetic $B = B_0\ \hat{z}$, write the value of $\oint_s \vec{B}.d\vec{a}$. **1**

(v) Which one of the following is an impossible electrostatic field?

$$\vec{E} = k[xy\hat{x} + 2yz\hat{y} + 3xz\hat{z}];$$

$$\vec{E} = k[y^2\hat{x} + (2xy + z^2)\hat{y} + 2yz\hat{z}].$$

Here k is a constant with the appropriate units. **2**

2. A thick spherical shell (inner radius 'a' and outer radius 'b') is made of a dielectric material with a frozen-in polarization $\vec{P}(r) = \frac{k}{r}\hat{r}$, where k is a constant and r is the distance from the center. Obtain
   (a) total surface charge at surfaces $r = a$, and $r = b$.
   (b) volume charge density ρ and the electric field at a point r so that $a < r < b$. **6**

3. (i) In a region of space $\vec{E} = 4x\hat{x} + 4y\hat{y} + 6\hat{z}$. Obtain the potential difference between points (1,1,1) and (2,2,2). Also calculate the charge enclosed within a sphere of radius 1m with its center at the origin. **3**

(ii) A spherical conductor of radius 'a' carries a charge Q. It is surrounded by a linear dielecric material of susceptibility χ_e, out to radius b. Find the energy of the configuration. **3**

4. (i) Starting with Biot-savart law, show that $\vec{\nabla}.\vec{B} = 0$. **3**

(ii) A long conducting cylinder of radius R carries a current density $\vec{J} = \hat{z}J_0\left\{1 - r^2/R^2\right\}$ along its axis. Obtain the magnetic field and vector potential at $r > R$. **4**

$k_{2z} =$ $(k_2^2 - k_{2x}^2)^{1/2}$

$k_2 = \frac{\omega}{c}\sqrt{2}$

$\frac{1}{\sqrt{2}}$ ... $2\sqrt{2}$ ... $2\omega^2/c^2$