Department of Mathematics

MTL 106 (Introduction to Probability Theory and Stochastic Processes)

Minor 2 (I Semester 2016 - 2017)

Time allowed: 1 hour | Max. Marks: 25

1. Let X be a random variable having an exponential distribution with parameter $\frac{1}{2}$. Let Z be a random variable having a normal distribution with mean 0 and variance 1. Assume that, X and Z are independent random variables.

   (a) Find the probability density function of $T = \frac{Z}{\sqrt{X}}$. (3 marks)

   (b) Compute $E(T)$ and $Var(T)$. (1 + 1 marks)

2. Reliability, denoted by $R(t)$, is defined as the probability that the component or system experiences no failures during the time interval 0 to t. An aircraft has four engines, each of which has an exponential distributed failure time with parameter λ.

   (a) For a successful flight at least two engines should be operating. Find the reliability $R(t)$ and expected lifetime of the aircraft. (1 + 2 marks)

   (b) Find the reliability and expected lifetime if the aircraft needs at least one operating engine on either side for a successful flight. (1 + 1 marks)

3. Suppose that A and B are two events associated with an experiment. Suppose that $P(A) > 0$ and $P(B) > 0$. Let the random variables X and Y be defined as follows:

   $$X = 1 \text{ if A occurs and 0 otherwise}$$
   $$Y = 1 \text{ if B occurs and 0 otherwise}$$

   Show that $\rho_{XY} = 0$ implies that X and Y are independent. (4 marks)

4. (a) State and prove Chebyshev's inequality. (1 + 2 marks)

   (b) Suppose that the number of customers who visit SBI, IIT Delhi on a Saturday is a random variable with $\mu = 75$ and $\sigma = 5$. Find the lower bound for the probability that there will be more than 50 but fewer than 100 customers in the bank? (2 marks)

5. Suppose that $X_i, i = 1, 2, \ldots, 450$ are independent random variables, each having a distribution $N(0, 1)$. Evaluate $P(X_1^2 + X_2^2 + \ldots + X_{450}^2 > 495)$ approximately.
   ($\Phi(2) = 0.9772$, $\Phi(1.5) = 0.9452$) (4 marks)

6. Consider the random telegraph signal, denoted by $X(t)$. At time $t = 0$, the signal $X(t)$ start with equal probability for the two states, i.e., $P(X(0) = 0) = P(X(0) = 1) = 1/2$, and let the switching times be decided by a Poisson process $\{Y(t), t \geq 0\}$ with parameter λ independent of $X(0)$. At time t, the signal $X(t) = \frac{1}{2}\left(1 - (-1)^{X(0)+Y(t)}\right), t > 0$. Write the state space (S) and the parameter space (T) of the stochastic process $\{X(t), t \in T\}$.

   (1 + 1 marks)