Electrical Engineering, IIT Delhi
Computer communications networks (ELL 785)
Final-term Examination

Duration: 2 hour

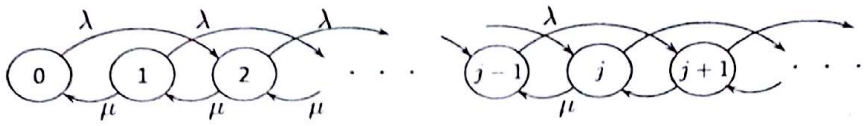
Nov. 24, 2016
Instructor: Jun B. Seo

Name Entry number

- This examination consists of **four(4)** problems with one bonus. Check that you have a complete copy of **four(4)** pages.
- Maximum attainable mark is 100.
- Justify your answers clearly.

1 (27)	2 (13)	3 (39)	4 (21)	sum

1. [27] A pair of customers, i.e., exactly two customers at a time, arrive based on Poisson process with rate λ. However, customers are served one by one and the service time of each customer is exponentially distributed random variable with mean $1/\mu$. The state transition diagram is shown below. Let π_j denote the steady state probability that the system has j customers.



(a) [6] Find three global balance equations for states $j = 0$, 1, and $j \geq 2$.

(b) [10] Find the probability generating function for π_j.

(c) [5] Find the probability that the system is empty, i.e., π_0.

(d) [6] Find the mean number of customers in the system.

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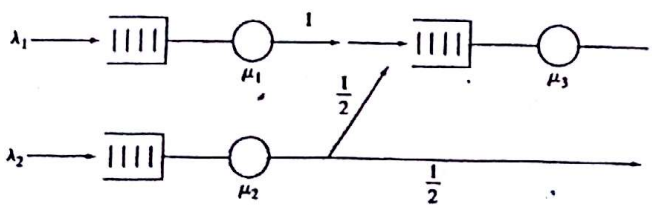
2. [13] Consider an opening queueing network consisting of three queues as shown below.



(a) [4] Find an expression of the average number of customers in the network.

(b) [5] If $\mu_1 = 0.7$, $\mu_2 = 0.9$, and $\mu_3 = 0.8$, draw a region for all possible pairs of λ_1 and λ_2 such that three queues are stable at the same time. Specify the region exactly.

(c) [4] Find the maximum possible throughput, while all queues are stable?

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3. [39] Suppose that a base station has two antennas such that it can decode maximally two packets transmitted at a time over the uplink. Time is divided into slots of equal size. At each slot, if there are n backlogged packets, each of them is transmitted with probability p. In addition, the probability that k packets join the backlog (without being transmitted immediately) follows a Poisson process with mean rate λ (packets/slot).

   (a) [9] When there are n backlogged users at time t, find the expression for $P_{n,n+i}$ for $-n \leq i < \infty$.

   (b) [9] Under what condition is the Markov chain positive recurrent? Express it in terms of p, λ, and n.

   (c) [4] What is the p of minimizing the drift?

   (d) [4] Find the maximum of λ allowed (maximum throughput) such that the system is stable. In this question, use the solution in (c) while assuming $n \to \infty$.

   (e) [4] Estimate the maximum throughput if the number of antennas is M.

   (f) [9] Assume that the number of backlogged packets follows a Poisson distribution with mean n_t at time t. Find an update rule for n_{t+1} when the system observes two successful packets.

4. [21] Persons arrive at a Xerox machine according to Poisson process with mean rate one per minutes. The number of copies to be made by each person is uniformly distributed between 1 and 10. Each copy requires 3 sec. Find the average system response time when:

   (a) [4] Each person uses the machine on a first-come first-serve basis

   (b) [5] Persons with no more than 2 copies to make have nonpreemptive priority: Response time type 1 and 2.

   (c) [5] Persons with no more than 2 copies to make have preemptive resume priority

   (d) [7] Repeat (a) when each copy requires exponentially distributed random time with mean 3 sec.