Quiz 6
(15 min, Total: 15)

Computer communications networks (ELL785)

Nov. 8, 2016
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Name: Entry number:

M/M/4

1. [15] IIT Delhi has four tennis courts of hard surface. People (always in pairs) arrive at the courts at a Poisson rate with mean α_1 (pairs/min) and use a court for an exponentially distributed time with mean 40 (min). After playing tennis, some of them go to cafeteria with probability $p = 0.4$, while others leave. At cafeteria, a pair of students who do not come from tennis court arrive based on Poisson process with mean α_2 (pairs/min). In cafeteria, there are eight tables and those who find all the tables occupied shall wait. Each pair of students stay at the table for exponentially distributed random times with mean 60 (min). Students in the cafeteria will go back to tennis court in a pair with probability $q = 0.3$ after enjoying some snack, while the rest shall leave.

   (a) [7] Find all pairs of α_1 and α_2 such that both queueing systems are stable.

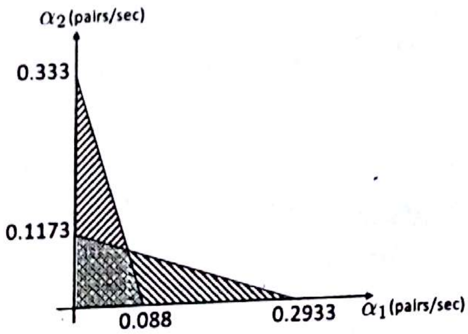
   (b) [8] Find the joint probability that there are n_1 pairs in the tennis courts and n_2 pairs in the cafeteria.

(a) We have first $\lambda_1 = \alpha_1 + q\lambda_2$ and $\lambda_2 = \alpha_2 + p\lambda_1$. We can rearrange
$\lambda_1 = \frac{1}{1-pq}(\alpha_1 + q\alpha_2) = 1.1364(\alpha_1 + 0.3 \cdot \alpha_2) = 0.1 \rightarrow \alpha_1 + 0.3 \cdot \alpha_2 = 0.088$ and
$\lambda_2 = \frac{1}{1-pq}(p\alpha_1 + \alpha_2) = 1.1364(0.4 \cdot \alpha_1 + \alpha_2) = 0.1333 \rightarrow 0.4 \cdot \alpha_1 + \alpha_2 = 0.1173.$

$\mu = \frac{1}{40}$

$m\mu = \frac{1}{10}$

$\lambda_1 < m\mu \; (0.1)$



$m\mu_2 = \frac{8}{60} = \frac{2}{15}$

(b) Let $P_i(n_i)$ be the probability of n_i pairs being in the system i. We can write first

$$P_1(n_1) = \rho_1^{\max(0,n_1-4)} \frac{a_1^{\min(4,n_1)}}{\min(4,n_1)!} \quad \text{and} \quad P_2(n_2) = \rho_2^{\max(0,n_2-8)} \frac{a_2^{\min(8,n_2)}}{\min(8,n_2)!},$$

which imply $P_i(0) = 1$. Note that $a_1 = \lambda_1/\mu_1$, $\rho_1 = \lambda_1/(4\mu_1)$, $a_2 = \lambda_2/\mu_2$, $\rho_2 = \lambda_2/(8\mu_2)$. Then, $\pi(n_1, n_2)$ is expressed as

$$\pi(n_1, n_2) = \frac{P_1(n_1)P_2(n_2)}{G},$$

where G is obtained from $\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty} \pi(n_1, n_2) = 1$.

$\lambda_2 = \alpha_2 + p(\alpha_1 + q\lambda_2)$?

$= \alpha_2 + p\alpha_1 + pq\lambda_2$

$\Rightarrow \lambda_2 = \frac{1}{1-pq}(\alpha_2 + p\alpha_1)$

$\lambda_1 = \frac{1}{1-pq}(\alpha_1 + q\alpha_2)$

~~$p\lambda_1 = p\alpha_1 + pq\lambda_2$~~

~~$q\lambda_2 = q\alpha_2 + pq$~~

The G is further calculated as

$$G = \left[\sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \rho_1^{\max(0,n_1-4)} \rho_2^{\max(0,n_2-8)} \frac{a_1^{\min(4,n_1)}}{\min(4,n_1)!} \frac{a_2^{\min(8,n_2)}}{\min(8,n_2)!} \right]$$

$$= \left[\sum_{n_1=0}^{\infty} \rho_1^{\max(0,n_1-4)} \frac{a_1^{\min(4,n_1)}}{\min(4,n_1)!} \sum_{n_2=0}^{\infty} \rho_2^{\max(0,n_2-8)} \frac{a_2^{\min(8,n_2)}}{\min(8,n_2)!} \right].$$

We know from $M/M/C$ queue: $\sum_{n_1=0}^{\infty} \rho_1^{\max(0,n_1-4)} \frac{a_1^{\min(4,n_1)}}{\min(4,n_1)!} = \left[\sum_{i=0}^{3} \frac{a_1^i}{i!} + \frac{a_1^4}{4!} \frac{1}{1-\rho_1} \right]$. Then, G is rewritten as

$$G = \left[\sum_{i=0}^{3} \frac{a_1^i}{i!} + \frac{a_1^4}{4!} \frac{1}{1-\rho_1} \right] \left[\sum_{i=0}^{7} \frac{a_2^i}{i!} + \frac{a_2^8}{8!} \frac{1}{1-\rho_2} \right].$$